

Self-Organized Criticality, Optimization and Biodiversity

Roberto N. Onody* and Paulo A. de Castro†

*Departamento de Física e Informática, Instituto de Física de São Carlos,
Universidade de São Paulo, C.P.369, 13560-970 São Carlos-SP, Brazil*

By driven to extinction species less or poorly adapted, the Darwinian evolutionary theory is intrinsically an optimization theory. We investigate two optimization algorithms with such evolutionary characteristics: the Bak-Sneppen and the Extremal Optimization. By comparing their mean fitness in the steady state regime, we conclude that the Bak-Sneppen dynamics is more efficient than the Extremal Optimization if the parameter τ is in the interval $[0, 0.86]$. The determination of the spatial correlation and the probability distribution of the avalanches show that the Extremal Optimization dynamics does not lead the system into a critical self-organized state. Trough a discrete form of the Bak-Sneppen model we argument that biodiversity is an essential requisite to preserve the self-organized criticality.

Keywords: Extremal optimization; Bak-Sneppen model; self-organized criticality

I. INTRODUCTION

By all that we know nature evolves in a self-organized critical state [1]. One of the most fundamental characteristics of a system in a self-organized critical state (SOC) is to exhibit a stationary state with a long-range power law decay of both spatial and temporal correlations [2]. Power law is a very abundant behavior appearing either in natural phenomena such as the light emitted from quasars, the earthquakes intensities, the water level of the Nile river or as a direct result of human activities like the distribution of cities by size, the repetition of words in the Bible and in traffic jams.

Self-organized critical systems evolve to the complex critical state without the interference of any external agent - there is no tuning parameter. The prototypical example of SOC is a pile of sand [2]. Usually, the self-organized state is attained only after a very long period of transient. Last but not least, a minor change in the system can cause colossal instabilities called avalanches. Intermittent bursts of activity separating long periods of quiescence is called punctuated equilibrium. Gould and Eldredge conjectured that the biological evolution in our planet is under the auspices of this kind of mechanism [3].

The evolution of the living beings is basically governed by the theory of natural selection. One model specially tailored to represent the co-evolutionary activities of the species is the Bak-Sneppen model (BS). In this model [4], each species occupies a site i of a lattice and has associated a fitness value λ_i between 0 and 1 (randomly drawn from a uniform distribution). At each time step, the species with the smallest fitness as well as its nearest neighbors are selected to replace their fitness with new random numbers. In one dimension, after a long transient time, almost all species have fitness larger than the critical value 0.67 [4].

Recently, inspired by natural processes, some heuristic optimization techniques have been proposed: genetics algorithms [5], simulated annealing [6] and extremal optimization [7]. The latter, the extremal optimization method (EO), is the most efficient since it brings the system faster and closer to its ground state. In brief words, this method consists of the following rules: 1) a fitness λ_i with values between 0 and 1 (randomly chosen from a uniform distribution) is associated with each site i of a lattice with N points; 2) all the lattice sites are increasingly ranked according to their fitness (the site with the worst fitness is of rank 1); 3) a site of rank k ($1 \leq k \leq N$) is selected with probability $P(k) \sim k^{-\tau}$ (τ is an arbitrary real positive number) and its corresponding variable λ_i is changed to λ'_i ; 4) repeat at step 2) as long as desired.

We observe that, differently from what happens with the Bak-Sneppen dynamics, the EO dynamics has neither a co-evolutionary feature (the extinction of one species has no influence on its neighbors) nor has the exact (Darwinian) characteristic of the elimination of the worst adapted species. In this sense, we can say that the BS algorithm is a coarse grained description of the biological evolution adopted by nature while the EO algorithm represents an optimized dynamics created by man. In this paper, we compare the efficiencies (measured by their mean fitness in the steady state) of the EO and the BS dynamics. For the EO dynamics, we show that the spatial distribution is constant meanwhile the distribution of avalanches has an exponential decay. Using a discrete form of the BS model we argue that variability of species is an essential requisite to keep self-organized criticality.

II. SIMULATIONS

To compare both dynamics, we simulated the BS and EO algorithms up to $1.1 \cdot 10^9$ runs on a one dimensional ring with $N=4001$ sites. To guarantee that the stationary regime has been achieved, we discarded the first $1.0 \cdot 10^8$ runs as the transient time. Time averages were then taken over the remaining steps. Figure 1 shows the aver-

*Electronic address: onody@ifsc.usp.br

†Electronic address: pac@ifsc.usp.br

age frequency of the fitness λ . Clearly, for $\tau = 0.05$ the EO behaves like a random walk having an almost uniform and constant fitness distribution. At $\tau = 0.5$ (1.0) the distribution is an increasing linear (exponential) function of λ . For the BS dynamics, however, the distribution has the form of a step function with a discontinuity at the critical point $\lambda_c \sim 0.67$. This critical point exists in all regular geometries or exponential networks [8, 9, 10], but not in scale-free networks [11]. The presence of this critical point in the BS algorithm is the first sign that a critical self-organized state has been developed. For the EO there is no such a point.

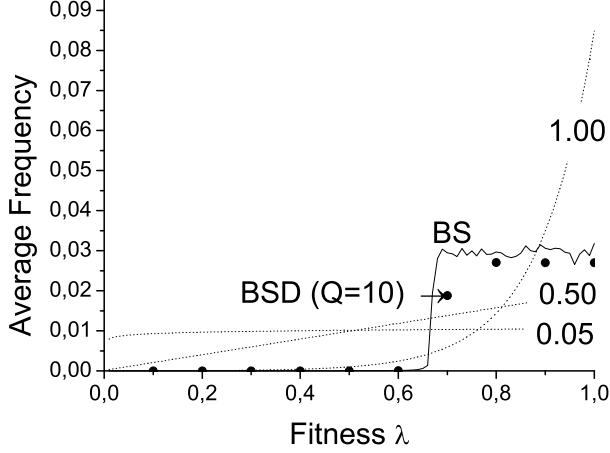


FIG. 1: The frequency of the fitness λ averaged on time after the stationary regime has been reached. The full line corresponds to the BS algorithm and exhibits a discontinuity at the critical point $\lambda_c \sim 0.67$. The dotted lines are the EO algorithm with $\tau = 0.05, 0.50$ and 1.00 . The points represent the discrete BSD algorithm with only 10 possible discrete fitness values (see the text).

To measure the algorithm's efficiencies, we plotted in Fig.2 the mean fitness obtained in the steady state regime. For the BS dynamics the mean fitness is 0.83. This mean fitness corresponds to an EO with $\tau = 0.86$. At $\tau = 1.5$ the mean fitness of the EO dynamics is approximately 0.99. This means that for higher values of τ , the EO algorithm leads to an utopian society where only one and perfect species survives. This limit corresponds to the simplified toy model proposed by K. K. Yee [12] in the context of law's evolution in the judicial system. For τ in the interval $[0, 0.86]$, the BS surpasses EO.

As we pointed out before, while BS is a co-evolutionary dynamics the EO dynamics is only evolutionary. The species in the EO do not interact. Comparison between co-evolutionary and evolutionary performances have already been done in the context of cellular automata [13]. To investigate the main differences between the evolutionary (EO) and co-evolutionary (BS) dynamics, we studied their spatial correlation dependence. Let $D(x)$ be the probability distribution of the distance x between two subsequent extinct (or mutated) species. From the

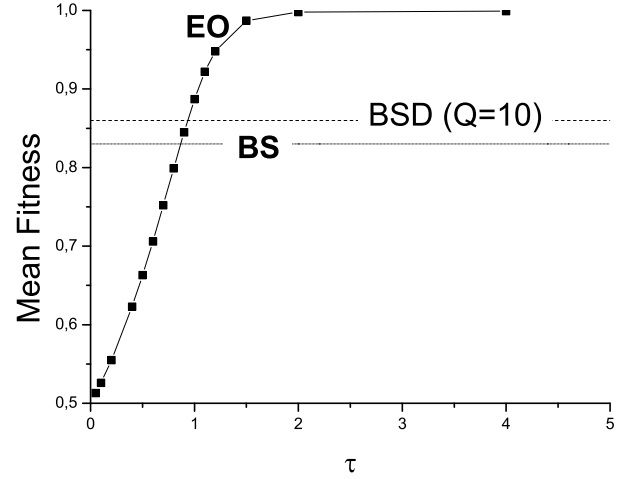


FIG. 2: The mean fitness of the EO dynamics for various values of τ . The BS (BSD) algorithm has a mean fitness value of 0.83 (0.86) which corresponds to $\tau = 0.86$ (0.95) in the EO curve.

Fig. 3, it is clear that the EO dynamics does not show a critical self-organized behavior. Instead of a power law, its spacial correlation is of infinite range - the probability distribution is constant no matter what is the distance between two subsequently modified species. For the BS dynamics, we find the well known power law dependence $D(x) \sim x^{-3.23 \pm 0.02}$ [14].

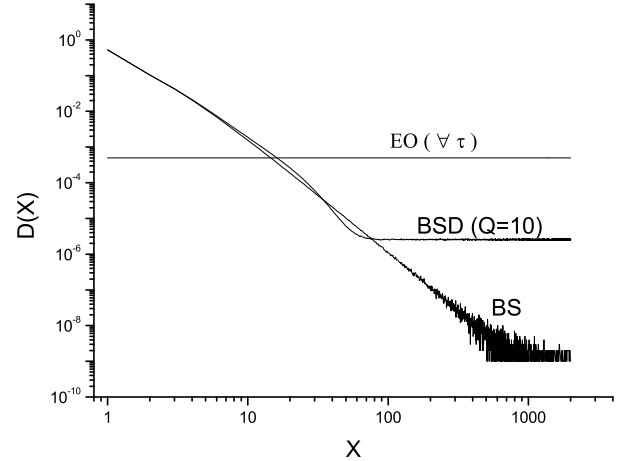


FIG. 3: The EO algorithm has a constant distribution probability $D(x)$ which is independent of the τ value. The BS algorithm shows a power law decay while the BSD has a mixed behavior.

Another important difference between the two dynamics is the complete absence of the punctuated equilibrium in the EO algorithm. One way to check out the existence of the punctuated equilibrium is to measure the probability distribution $P(A)$ of the avalanches with size A . The size A of an avalanche is defined as been the number

of subsequent time steps with at least one fitness value below a critical threshold λ_c . This critical point doesn't exist for the EO (Fig. 1). For the BS algorithm the distribution decays as $P(A) \sim A^{-1.07 \pm 0.01}$ at $\lambda_c = 0.67$ [14]. In the EO algorithm, on the other hand, the decay is exponential with a characteristic avalanche size $A_c(\lambda_c, \tau)$ depending on the choices made for λ_c and τ .

III. CONCLUSIONS

We conclude that although the efficiency of the EO algorithm may exceed, under certain circumstances (if $\tau > 0.86$), that of the BS dynamics, it is accompanied by three undesirable characteristics: the spatial correlation between the species is constant, i. e., it is independent of their distances, there is an external free parameter τ to be adjusted by hand and the punctuated equilibrium mechanism is lost. The punctuated equilibrium seems to be a very productive form found by nature to innovate species without the intervening of climatic changes or meteors destruction.

We have learned that the EO dynamics does not conduct the system to a critical self-organized state. However, we would like to point out that even the BS can loose its SOC characteristics and, amazingly, in a very easy and quick manner. Suppose that, instead a continuous and uniform fitness distribution in the interval $[0, 1]$, only some discrete values are now possible. To simplify, assume that the fitness can only have Q equally spaced values, i. e., $\lambda = m/Q$, with $m = 1, 2, \dots, Q$. Practically this means, that for some reason, the system's biodiversity has decreased. Due to the discreteness, there will be

an enormous number of species carrying the same (worst) fitness value. Which species should then we choose? The simplest solution is to put all those species in a list and to draw one of them. We will call this dynamics as Bak-Sneppen with draw (BSD). In the Fig.1 we plotted the case $Q = 10$ and observe that, like in the EO dynamics, there is not a critical threshold λ_c . The mean fitness is 0.86 (Fig.2), a value which is a little bit greater than that of the standard BS. The curve of the spatial probability distribution $D(x)$ (see Fig.3) is even more interesting. It shows that the BSD dynamics is of a mixed kind: it behaves like the BS for small distances and like the EO for large distances. So, the BSD dynamics does not retain the self-organized criticality characteristic. Just like in nature, biodiversity plays a fundamental role in the evolutionary theoretical models: without it self-organized criticality is not possible. For higher plants and animals the conventional explanations of biodiversity are habitat heterogeneity, predation pressure and niche differentiation. For microscopic organisms, however, the high biodiversity found (even in uniform environments) is not completely understood and it is called "the paradox of the plankton". Theoretically, such difficulties can be surmounted by incorporating a noise η into the fitness $\lambda = m/Q + (1 - 2r)\eta$ (where r is a random number in the interval $[0, 1]$ and generated from a uniform distribution). Even for η as small as 10^{-12} the SOC characteristic is preserved [15]. The noise can be interpreted as the presence of sub-species.

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- [1] P. Bak, *How Nature Works* (Springer, New York, 1996).
 - [2] P. Bak, C. Tang and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).
 - [3] S. J. Gould, Paleobiology **3**, 135 (1977); N. Eldredge and S. J. Gould, Paleobiology **3**, 115 (1977).
 - [4] P. Bak and K. Sneppen, Phys. Rev. Lett. **71**, 4083 (1993).
 - [5] J. Holland, *Adaptation in Natural and Artificial Systems* (University of Michigan Press, Ann Arbor, 1975).
 - [6] S. Kirkpatrick, C. D. Gellat and M. P. Vecchi, Science **220**, 671 (1983).
 - [7] S. Boettcher and A. G. Percus, Phys. Rev. Lett. **86**, 5211 (2001); Artif. Intellig. **119**, **275** (2000).
 - [8] T. S. Ray and N. Jan, Phys. Rev. Lett. **72**, 4045 (1994).
 - [9] S. Boettcher and M. Paczuski, Phys. Rev. Lett. **76**, 348 (1996).
 - [10] K. Christensen, R. Donangelo and B. Koiller, Phys. Rev. Lett. **81**, 2380 (1998).
 - [11] Y. Moreno and A. Vazquez, Europhys. Lett. **57**, 765 (2002).
 - [12] K. K. Yee, e-print nlin.AO/0106028.
 - [13] L. Pagie and M. Mitchell, International Journal of Computational Intelligence and Applications **2**, 53 (2002).
 - [14] M. Paczuski, S. Maslov and P. Bak, Phys. Rev. E **53**, 414 (1996).
 - [15] R. N. Onody and P. A. de Castro, *Optimization and Self-Organized Criticality in a Magnetic Systems*, to be published in Physica A.